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Possible Commensurabilities Among Pairs of Extrasolar Planets

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ABSTRACT
We investigate the possible commensurabilities to be expected when two protoplanets in the Jovian mass range, gravitationally interacting with each other and an external protoplanetary disc, are driven by disc induced orbital migration of the outer protoplanet into a commensurability which is then maintained in subsequent evolution. We find that for a variety of protoplanet masses and typical protoplanetary disc properties, as well as the setting up of 2:1 commensurabilities of the type recently observed in GJ876, 3:1 commensurabilities are often formed, in addition to 4:1, 5:1, and 5:2 commensurabilities which occur less frequently. The higher order commensurabilites are favoured when either one of the planets is massive, or the inner planet begins with a significant orbital eccentricity. Detection of such commensurabilities could yield important information relating to the operation of protoplanet disc interactions during and shortly post formation.

Key words: giant planet formation - extrasolar planets - - orbital migration - resonance-protoplanetary discs - stars: individual: GJ876

1 INTRODUCTION

The recent discovery of a pair of extrasolar giant planets orbiting in a 2:1 commensurability around GJ876 (Marcy et al 2001) has raised interesting questions about the post-formation orbital evolution of this system. Assuming the system did not form in its currently observed state, the existence of the commensurability indicates that disc induced orbital migration is likely to have occurred so as to gradually reduce the planetary orbital separation until resonance was established.

Simulations of two planets in the Jovian mass range interacting with a disc have been performed by Kley (2000) and Bryden et al (2000). The latter authors found a tendency for the two planets to open up gaps in their local vicinity, and for material between the two planets to be cleared, ending up interior to the inner planet orbit or exterior to the outer planet orbit, such that both planets orbit within an inner cavity.

Snellgrove, Papaloizou & Nelson (2001) (hereafter paper I) performed a simulation of a system consisting of a primary star with two planets moving under their mutual gravitational attraction and forces produced by tidal interaction with an externally orbiting gaseous disc. Angular momentum exchange caused the outermost planet to migrate inwards until a 2:1 commensurability with the inner planet was reached. The subsequent dynamical interaction then resulted in the planets migrating inwards together maintaining the commensurability.

The simulation results in paper I could be well matched with those of a simple N body integration procedure used below, which incorporated simple prescriptions for the migration and eccentricity damping of the outer planet due to interaction with the disc. Using this we investigate the range of commensurabilities that might be expected in two planet systems as a consequence of protoplanet disc interactions occurring in a standard protoplanetary disc model, where we assume a fixed migration time and consider various eccentricity damping rates.

Depending on the nature of the interaction between the disc and the outer planet, as well as the details of the resonance into which the planets enter, we distinguish between four types of orbital evolution in the resonant migration phase which we denote as types A – D. We denote the orbital elements of the outer planet with a subscript '1' and the inner planet with subscript '2'. Note that we find the evolution of the system depends on the details of the resonance into which the planets become locked. For higher order resonances such as 3:1 and 4:1, commensurabilities may be maintained in which differing resonant angles librate such that they do not cover the full $(0, 2\pi)$ domain (see for example Murray & Dermott 1999 for an extensive discussion). For a $p : q$ commensurability, we monitor the resonant angles defined by

$$\phi_{p,q,k} = p\lambda_1 - q\lambda_2 - p\varpi_1 + q\varpi_2 + k(\varpi_1 - \varpi_2). \tag{1}$$

Here $\lambda_1, \lambda_2, \varpi_1$ and ϖ_2 denote the mean longtitudes and longtitudes of periapse for the planets 1 and 2 respectively. The positive integers p and q satisfy $p > q$, and there are $p - q + 1$ possible values of the positive integer k such that $q \le k \le p$. Although there are $p - q + 1$ corresponding angles, no more than two can be linearly independent. This means that if libration occurs, either all librate or only one librates. Both situations occur in our integrations.

Type A: Eccentricities increase during migration until they reach steady state equilibrium values, with possibly small oscillations superposed. After this migration continues in a self-similar manner and all the resonant angles defined above are in libration.

Type B: Eccentricities increase as the migration proceeds, until $e_1 \ge 0.2$ at which point we estimate the outer planet enters the outer disc. Our simple model breaks down at this point. Large values of e_1 arise when the eccentricity damping rate is small. As in type A resonant migration all the resonant angles considered go into libration. The difference is that e_1 exceeds 0.2 before a steady state can be reached. An experimental model of the evolution of a two planet system corresponding to type B behaviour, and based on our knowledge of the migration of an eccentric planet interacting with a protoplanetary disc is presented in section 3.4.

Type C: For higher initial values of e_2, we find a mode of evolution in which e_1 and e_2 rise continuously during the migration phase, even for efficient damping of eccentricity. The issue of the outer planet entering the disc again arises in type C migration. The primary difference between type C evolution and types A and B is that only one of the resonant angles defined above is found to librate for the same apparent commensurability, so that the evolution differs.

Type D: This mode of resonant migration corresponds to small values of $e_1 < 0.2$ being maintained for the outer planet, whilst the eccentricity of the inner planet grows to attain values close to unity. In this respect it differs from the other types of migration. This mode of evolution is related to that described by Beust & Morbidelli (1996) when discussing the generation of star grazing comets in the β Pictoris system. All the resonant angles librate in type D migration but with large amplitude.

2 CONSERVATION OF ENERGY AND ANGULAR MOMENTUM

We consider the consequences of a simple application of the conservation of energy and angular momentum. In the case when a near steady state for the orbital eccentricities is attained, a relationship between the orbital eccentricities and the circularization and migration rates induced by the disc is obtained for the case of type A migration.

We consider two planets with masses m_1, m_2, osculating semi-major axes a_1, a_2, and eccentricities e_1, e_2. These orbit a central mass M_*. The total angular momentum is

$$J = J_1 + J_2 = m_1\sqrt{GM_* a_1(1-e_1^2)} + m_2\sqrt{GM_* a_2(1-e_2^2)} \tag{2}$$

and the energy E is

$$E = -\frac{GM_* m_1}{2a_1} - \frac{GM_* m_2}{2a_2} \tag{3}$$

We assume resonant self-similar migration in which a_2/a_1, e_1, and e_2 are constant. Then conservation of angular momentum gives

$$\frac{dJ}{dt} = J_1 \frac{1}{2a_1}\frac{da_1}{dt}\left(1 + \frac{m_2\sqrt{a_2(1-e_2^2)}}{m_1\sqrt{a_1(1-e_1^2)}}\right) = -T \tag{4}$$

and conservation of energy gives

$$\frac{dE}{dt} = \frac{GM_* m_1}{2a_1^2}\frac{da_1}{dt}\left(1 + \frac{m_2 a_1}{m_1 a_2}\right) = -\frac{n_1 T}{\sqrt{1-e_1^2}} - D, \tag{5}$$

where $n_1 = \sqrt{GM_*/a_1^3}$ and we suppose there is a tidal torque $-T$ produced by the disc which acts on m_1. In addition we suppose there to be an associated tidally induced orbital energy loss rate which is written as $n_1 T/\sqrt{1-e_1^2} + D$, with $D \equiv (GM_* m_1 e_1^2)/(a_1(1-e_1^2)t_c)$. Here t_c is the circularization time of m_1 that would apply if the tidal torque and energy loss rate acted on the orbit of m_1 with m_2 being absent. In that case we would have $de_1/dt = -e_1/t_c$. A migration time t_{mig} can be defined through $T = m_1\sqrt{GM_* a_1(1-e_1^2)}/(3t_{mig})$. This is the time for n_1 to increase by a factor of e if m_2 was absent and the eccentricity e_1 was fixed. Note that t_c and t_{mig} are determined by the disc planet tidal interaction and may depend on e_1. By eliminating $\frac{da_1}{dt}$ from (5) and (4) we can obtain a relationship between e_1, e_2, t_c, and t_{mig} in the form

$$e_1^2 = \frac{t_c\left(1 - e_1^2 - \frac{(1-e_2^2)^{1/2}(1-e_1^2)^{1/2}}{a_2^{-3/2}a_1^{3/2}}\right)}{3t_{mig}\left(\frac{m_1 a_2}{m_2 a_1} + \frac{a_2^{3/2}(1-e_2^2)^{1/2}}{a_1^{3/2}(1-e_1^2)^{1/2}}\right)} \tag{6}$$

We comment that this is general in that it depends only on the conservation laws and applies for any magnitude of eccentricity. However, it's derivation did assume self-similar migration with equilibrium eccentricities and so it does not apply in non equilibrium situations for which the eccentricities grow continuously. Note too that for a 2:1 commensurability for which $(a_1/a_2)^{3/2} = 2$, (6) reduces in the case $e_i^2 \ll 1$ to the expression given in paper I which was obtained by analysis of the perturbation equations directly:

$$e_1^2 = \frac{t_c m_2 a_1}{3t_{mig}(2m_1 a_2 + m_2 a_1)} \tag{7}$$

The above determines the eccentricity of the outer planet e_1 as a function of t_c and t_{mig}.

3 NUMERICAL CALCULATIONS

3.1 Model Assumptions and Physical Setup

The basic assumptions of our model are that the two planets orbit within the inner cavity of a tidally truncated disc that lies exterior to the outer planet. Tidal interaction with this disc causes inwards migration of the outer planet on a time scale of t_{mig}, and also leads to eccentricity damping of the outer planet on a time scale of t_c. We assume that the inner edge of the disc is such that an eccentricity of $e_1 \gtrsim 0.2$ will enable the outer planet to enter the disc, in basic agreement with the results of hydrodynamic simulations (e.g. Nelson et al 2000).

We have performed three-body orbit integrations using

a fifth-order Runge-Kutta scheme, and the Burlisch-Stoer method as an independent check (e.g. Press et al. 1993). A torque was applied to the outermost planet such that it migrated inwards on a time scale of $t_{mig} = 10^4$ local orbital periods and a damping force proportional to the radial velocity was applied in the radial direction. This has a fixed constant of proportionality such that for small e_1 and in the absence of disc torques, the eccentricity damps on a time scale of $t_c = 600\mathcal{N}$ local orbital periods with values of $\mathcal{N} = 1, 10, 100$ having been considered. We comment that we have adopted the simplification of neglecting any possible dependence of the damping force and excepting the calculation presented in section 3.4, t_{mig}, on e_1.

This procedure was shown to be capable of matching the results of a detailed simulation in paper I, and the value of t_{mig} adopted is consistent with that found from hydrodynamic simulations of protoplanetary discs interacting with protoplanets in the Jovian mass range (e.g. Nelson et al 2000). Consideration of the orbital parameters of GJ876 (see paper I) suggests $\mathcal{N} \sim 1$. However, there is some uncertainty in the functional dependence of the circularization rate of an orbiting protoplanet nonlinearly interacting with a tidally truncated protostellar disc as it would depend on the distance to and form of the disc edge (Goldreich & Tremaine 1980), and be sensitive to the presence of coorbital material which can lead to eccentricity damping through coorbital Lindblad torques (Artymowicz 1993).

For length scale we adopt a fiducial radius, R, which for simplicity is taken to be 1 AU. However, scaling invariance allows this to be scaled to some other value if required. Runs were typically started with the outer planet at radius $15.6R$ while the inner planet was started at radius $5R$. A larger initial separation would be inconsistent with our model assumption of there being two planets orbiting within a tidally cleared cavity. Simulations by Bryden et al. (2000) suggest that the time scale to clear such a cavity becomes comparable to the migration time if the ratio of planetary semimajor axies becomes much larger than 3. The outer planet was assumed, because of interaction with the disc, to be in a circular orbit while starting eccentricities $e_2 = 0.0, 0.05, 0.1, 0.2, 0.3$ were adopted for the inner planet. For protoplanet masses we have considered all permutations of 0.4, 1 and 4 Jupiter masses assuming the central star mass is 1 $M_\odot$.

3.2 Results

The results of our survey are shown in table 1. In general for the shortest circularization rates with $\mathcal{N} = 1$ and initial values of $e_2 \leq 0.1$, self-similar (type A) migration occurred. For larger values of $\mathcal{N}$, type B migration was usually the preferred outcome.

For larger initial values of e_2 and $\mathcal{N} = 1$, either type A or type C migration occurred. For larger values of $\mathcal{N}$, types A, B, C, or D were all found to occur, with a strong preference for types B and C. Only one example of type D migration arose in our simulations.

3.3 Initial $e_2 = 0$

For an inner planet on an initially circular orbit, a 2:1 commensurability normally resulted, except when one of the

m_1	m_2	$\mathcal{N}$	Resonance				
			$e_2 = 0$	0.05	0.1	0.2	0.3
4	4	1	3:1A	3:1A	3:1A	4:1A	3:1A
4	4	10	3:1B	4:1B	4:1B	3:1B	4:1C
4	4	100	3:1B	3:1B	3:1B	3:1B	5:1C
4	1	1	3:1A	3:1A	4:1A	3:1A	2:1A
4	1	10	3:1B	3:1B	3:1B	2:1A	4:1D
4	1	100	3:1B	3:1B	3:1B	3:1C	2:1B
4	0.4	1	3:1A	3:1A	3:1A	5:2A	4:1A
4	0.4	10	3:1A	3:1A	3:1A	3:1A	2:1A
4	0.4	100	3:1B	3:1B	3:1B	2:1B	2:1B
1	4	1	2:1A	3:1A	3:1A	3:1A	3:1C
1	4	10	3:1B	3:1B	3:1B	5:1C	5:1C
1	4	100	3:1B	3:1B	4:1B	5:1C	3:1B
1	1	1	2:1A	2:1A	3:1A	4:1C	2:1A
1	1	10	2:1B	2:1B	2:1B	4:1B	5:1B
1	1	100	2:1B	3:1B	4:1B	3:1B	5:1B
1	0.4	1	2:1A	3:1A	3:1A	2:1A	2:1A
1	0.4	10	2:1B	2:1B	3:1B	3:1B	3:1B
1	0.4	100	2:1B	3:1B	3:1B	2:1B	3:1B
0.4	4	1	2:1A	3:1A	3:1A	3:1C	3:1C
0.4	4	10	2:1B	3:1B	3:1B	5:1C	5:1C
0.4	4	100	3:1B	3:1B	4:1B	5:1C	5:2B
0.4	1	1	2:1A	2:1A	3:1A	2:1A	4:1C
0.4	1	10	2:1B	3:1B	2:1B	2:1B	5:2C
0.4	1	100	2:1B	5:2B	3:1B	5:2C	2:1B
0.4	0.4	1	2:1A	2:1A	3:1A	2:1A	2:1A
0.4	0.4	10	2:1B	2:1B	2:1B	3:1B	3:2B
0.4	0.4	100	2:1B	3:1B	2:1B	2:1B	2:1B

Table 1. This table indicates the outcome and commensurability attained when a pair of masses m_1, m_2 measured in Jupiter masses becomes resonantly coupled due to disc driven migration of the outer planet. The initial eccentricity e_2 and $\mathcal{N}$ are indicated, e_1 is always initated as zero. The letters A, B, C, and D in columns 4 – 8 indicate whether types A, B, C, or D evolution occurred.

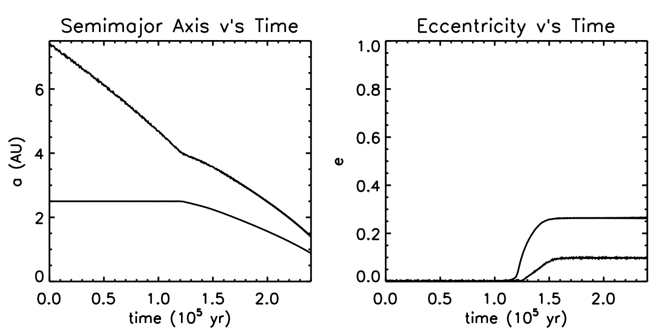

Figure 1. This figure shows the evolution of the planet semimajor axes and eccentricities for a pair of protoplanets with $m_1 = 1$, $m_2 = 1$, initial $e_2 = 0$, and $\mathcal{N} = 1$. This case underwent self-similar (type A) migration in 2:1 resonance after having attained equilibrium eccentricities. The outer planet is denoted by the upper line in the first panel and the lower line in the second panel. The unit of time is $10^5 (R/1\mathrm{AU})^{3/2}$ yr.

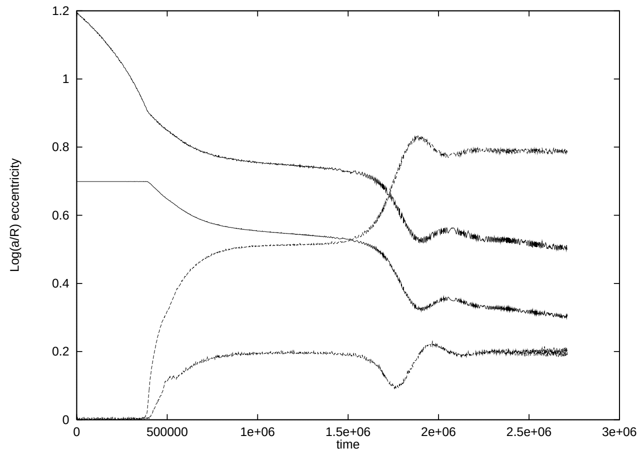

Figure 2. This figure shows the evolution of the protoplanet semimajor axes and eccentricities for a pair of protoplanets, each of 1 Jupiter mass with $\mathcal{N} = 100$ but with the migration rate of the outer planet modified such that it reverses for $e_1 > 0.2$. At small times, the upper two curves show $\log_{10}(a/R)$ while the lower two curves show e_1, e_2. The upper curve of the two denoting $\log_{10}(a/R)$ represents the outer planet, whereas at late times the lower curve of the two representing e_1, e_2 represents the outer planet. The unit of time is $(R/1\mathrm{AU})^{3/2}$ yr.

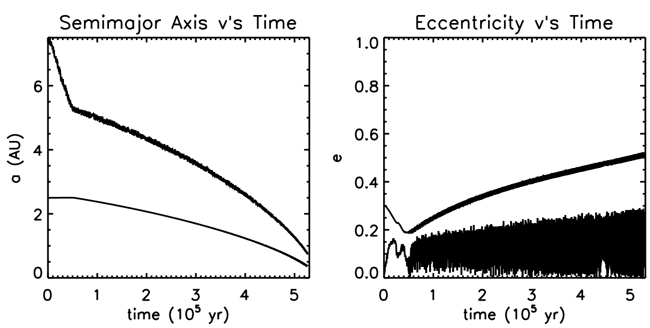

Figure 3. This figure shows the evolution of the protoplanet semimajor axes and eccentricities for a pair of protoplanets, with $m_1 = 1$, $m_2 = 4$, initial $e_2 = 0.3$, and $\mathcal{N} = 1$. This case provides an example of type C migration in 3:1 resonance. Here the eccentricities of both planets grew continuously as the system evolved, even with significant damping of e_1 by the disc. Note that our simple model breaks down for $e_1 > 0.2$. The outer planet is denoted by the upper line in the first panel and the lower line in the second panel. The unit of time is $10^5(R/1\mathrm{AU})^{3/2}$ yr.

masses was 4 Jupiter masses in which case a 3:1 commensurability could occur. Plots of the evolution of the semimajor axes and eccentricities of a Jupiter mass pair with $\mathcal{N} = 1$ which undergo self-similar migration are illustrated in figure 1. In this case steady equilibrium eccentricities are obtained that are consistent with equations (6) and (7). We found that for the adopted migration rate, the transition between a 2:1 and a 3:1 commensurability occured for m_1 in the range 2–3 Jupiter masses when m_2 was a Jupiter mass, with the transition mass being smaller for larger migration rates.

3.4 Initial $0 < e_2 \le 0.1$

When the inner planet was started with an initial eccentricity such that $0 < e_2 \le 0.1$, trapping in a 3:1 commensura-

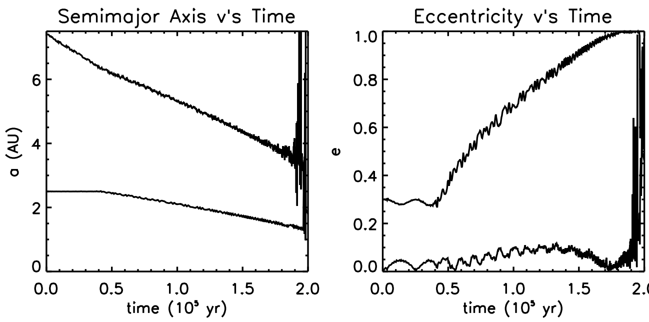

Figure 4. This figure shows the evolution of the protoplanet semimajor axes and eccentricities for a pair of protoplanets, with $m_1 = 4$, $m_2 = 1$, initial $e_2 = 0.3$, and $\mathcal{N} = 10$. This run provides an example of type D migration in 4:1 resonance. Here the eccentricity of the inner planet grows without limit during the evolution, eventually reaching $e_2 \simeq 1$, whereas the eccentricity of the outer planet remains relatively small. As e_2 approaches 1, a dynamical instability occurs, causing the planets to be scattered. The simplicity of our disc model does not allow this phase of evolution to be modeled accurately. The outer planet is denoted by the upper line in the first panel, and by the lower line in the second panel. The unit of time is $10^5(R/1\mathrm{AU})^{3/2}$ yr.

bility was more common and could occur even for the lowest mass pairs. Also a few cases of trapping in a 4:1 commensurability and one 5:2 commensurability were found. It was observed that the mode of migration in these cases usually corresponded to type A for $\mathcal{N} = 1$, and type B for $\mathcal{N} = 10$ or 100.

In order to investigate the potential outcome of a type B migration we considered a Jupiter mass pair with $\mathcal{N} = 100$ in which case the eccentricity of the outer planet grows to exceed 0.2. At this stage the assumed migration rate becomes inapplicable. In fact migration may reverse on account of penetration of the disc by the outer planet. In this context we note that a simulation of massive protoplanets by Papaloizou, Nelson & Masset (2001) indicated such a potential reversal for $e_1 > 0.2$ and linear torque calculations by Paploizou & Larwood (2000) suggested migration reversal for an embedded protoplanet with eccentricity exceeding a few times the disc aspect ratio. As an experiment we modified the disc induced migration rate of m_1 by a factor $(1 - (e_1/0.2)^2)$. The resulting evolution is shown in figure 2 This adjustment can stall the migration rate as e_1 approaches 0.2 and lengthen the lifetime of the system before a potential orbital instability occurs due to the development of large eccentricities in both planets. In fact in this case the eccentricity of the inner planet reached ~ 0.8 after $\sim 2 \times 10^6(R/1\mathrm{AU})^{3/2}$ y. The system was found to be subsequently stable for at least a similar time after a rapid disc removal.

Thus we emphasise that the projected lifetimes of the resonantly migrating systems discussed here are dependent on the nature of the disc interaction, and at present are very uncertain. However, they could approach the protoplanetary disc lifetime if the mode of evolution discussed in the preceding paragraph was to occur.

3.5 Initial $0.2 \leq e_2 \leq 0.3$

For initial values of e_2 in the range $0.2 \leq e_2 \leq 0.3$, it was observed that type C evolution became an important consideration, with only a single case of type D evolution being observed. For type C evolution, as in type B, the eccentricities of both planets were observed to grow without reaching equilibrium, until $e_1 > 0.2$ at which point our simple model breaks down. The long term evolution in this situation may resemble that described in figure 2. We illustrate the behaviour of type C evolution in figure 3.

The single case of type D evolution obtained is illustrated in figure 4, which should be compared with figure 3 since the only difference between these runs is the initial value of e_2 ($e_2 = 0.2$ in figure 3 and $e_2 = 0.3$ in figure 4). In this mode of evolution, the outer planet maintains a modest eccentricity while the eccentricity of the inner planet grows without limit. Although this mode of evolution appears to be rather rare based on the results of our calculations, it is interesting to note that it provides a method of generating very high eccentricities, such as that observed in the system HD80606 where the planetary eccentricity is $e \gtrsim 0.9$ (Naef et al. 2001).

Even for initially high values of e_2, we still find a tendency for lower mass pairs of planets to enter lower order resonances such as 2:1, 3:1 and even 3:2, whilst the higher mass pairs tend to occupy the higher order commensurabilities, even resulting in capture into 5:1 on a significant number of occassions.

3.6 The Case of GJ876

We have run a number of calculations to examine the putative early evolution of the observed system GJ876, which exists in a 2:1 commensurability. The minimum masses of the two planets, in units where the central star s of solar mass, are $m_1 = 6$ M$_J$and $m_2 = 1.8$ M$_J$. These parameters suggest that trapping in higher order resonance may have been expected rather than in 2:1. For initial conditions in which $m_1 = 6$, $m_2 = 1.8$, initial $e_2 = 0$, capture into 3:1 resonance is favoured. For initial $e_2 = 0.1$, 4:1 resonance results. However, in a scenario in which the planetary masses were smaller during the initial resonant capture, capture into 2:1 can occur. Simulations were performed with $m_1 = 1$, $m_2 = 1.8$, initial $e_2 = 0$ and 0.1, which all favour capture into 2:1. Assuming a mass for the outer planet of $m_1 = 3$ produces capture in 3:1.

Two possible conclusions to draw from this are that:
(1). resonant capture occurred in this system when the planets were of smaller mass, and that the mass of the outer planet increased due to accretion of gas from the disc.
(2). The planets formed sufficiently close together initially that capture into resonances of higher order than 2:1 was not possible.

4 SUMMARY AND DISCUSSION

In this paper we have considered two protoplanets gravitationally interacting with each other and a protoplanetary disc. The two planets orbit interior to a tidally maintained disc cavity while the disc interaction induces inward migration. We find that as well as 2:1 commensurabilities being formed, 3:1 commensurabilities are obtained in addition to 4:1, 5:1, and 5:2 commensurabilities which occur less frequently and usually for larger initial eccentricities of the inner planet. Possession of a small eccentricity ~ 0.1 before resonant locking takes place would favour 3:1 commensurabilities if migration over large separations has occurred. Initially near circular orbits would favour 2:1 commensurabilities except when one of the planets is significantly more massive than one Jupiter mass.

Up to now seven multiple planet systems have been detected with two showing a 2:1 commensurability (GJ876, HD82943 – see the website:
obswww.unige.ch/~udry/planet/new_planet.html). It is too early at present to establish the frequency of occurence of commensurabilities in multiple systems but they may be fairly common. If orbital migration has occured over a large scale in extrasolar planetary systems the future detection of additional commensurabilities of the type discussed here is to be expected. We also comment that in some cases a high eccentricity for the inner planet orbit may be produced through resonant migration. If, depending on the details of disc dispersal and interaction, a scattering followed by ejection of the outer planet occurs, a single planet remaining on an eccentric orbit may result. This will be a topic for future investigation.

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